Then $EF=\dfrac{2(a^2+ab-2ab\sin^2\gamma)}{(a+b)\sqrt{1-e^2\sin^2\gamma}}$.

$$\therefore\ EF=\frac{2a}{\sqrt{1-e^2\sin^2\gamma}}+\frac{2}{e\sqrt{ab}}\sqrt{1-e^2\sin^2\gamma}-\frac{2}{e\sqrt{ab}\sqrt{1-e^2\sin^2\gamma}}.$$

$$\therefore\int EFd\gamma=\frac{2}{e\sqrt{ab}}\{(ae\sqrt{ab}-1)F(e,\gamma)+E(e,\gamma)\}.$$

$$\therefore\ \varDelta=\frac{2}{\beta e\sqrt{ab}}\{(ae\sqrt{ab}-1)F_0^\beta(e,\gamma)+E_0^\beta(e,\gamma)\},\ b>a.$$

$$\varDelta=\frac{2}{\delta e\sqrt{ab}}\{(ae\sqrt{ab}-1)F_0^\delta(e,\gamma)+E_0^\delta(e,\gamma)\},\ b<a.$$

II. $\varDelta_1=2\int_0^{\theta'}\sqrt{a^2-b^2\sin^2\theta}\,d\theta\Big/\int_0^{\theta'}d\theta=\dfrac{2}{\theta'}\int_0^{\theta'}\sqrt{a^2-b^2\sin^2\theta}\,d\theta.$

Let $\theta=\frac{1}{2}\pi+\lambda$. $\therefore\ \theta'-\frac{1}{2}\pi=\lambda$ to $-\frac{1}{2}\pi=\lambda$.

$$\therefore\ \varDelta_1=\frac{2}{\theta'}\int_{-\frac{1}{2}\pi}^{\theta'-\frac{1}{2}\pi}\sqrt{a^2-b^2\cos^2\lambda}\,d\lambda=\frac{2}{\theta'}\int_{-\frac{1}{2}\pi}^{\theta'-\frac{1}{2}\pi}\sqrt{b^2\sin^2\lambda-(b^2-a^2)}\,d\lambda$$

$$=\frac{2\sqrt{(b^2-a^2)}}{\theta'}\int_{-\frac{1}{2}\pi}^{\theta'-\frac{1}{2}\pi}\sqrt{\frac{b^2}{b^2-a^2}\sin^2\lambda-1}\,d\lambda$$

$$=\frac{2\sqrt{(b^2-a^2)}}{\theta'}H_{-\frac{1}{2}\pi}^{\theta'-\frac{1}{2}\pi}\left(\frac{b}{\sqrt{b^2-a^2}},\lambda\right),\ b>a.$$

$$\varDelta_1=2\int_0^{\frac{1}{2}\pi}\sqrt{a^2-b^2\sin^2\theta}\,d\theta\Big/\int_0^{\frac{1}{2}\pi}d\theta=\frac{4a}{\pi}E_0^{\frac{1}{2}\pi}\left(\frac{b}{a},\theta\right),\ b<a.$$

45. Proposed by J. C. WILLIAMS, Boston, Massachusetts.

At the end of the fifth inning the base ball score stands 7 to 9. What is the probability of winning for either team?

Solution by J. SCHEFFER, A. M., Hagerstown, Maryland.

From the stated score we are able to estimate the respective skill of the two teams, and their respective probabilities of winning the game.

The respective probabilities are $\frac{7}{16}$ and $\frac{9}{16}$. We have now to find the probabilities of either team winning *at least* 3 games out of 4, granting, of course, 9

innings to be played. These probabilities are respectfully, $(\frac{7}{16})^4+4(\frac{7}{16})^3\cdot\frac{9}{16}=\frac{14749}{65536}$, and $(\frac{9}{16})^4+4.(\frac{9}{16})^3\cdot\frac{7}{16}=\frac{26973}{65536}$.

46. Proposed by J. A. CALDERHEAD, M. Sc., Professor of Mathematics in Curry University, Pittsburg, Pennsylvania.

Four men starting from random points on the circumference of a circular field and traveling at different rates, take random straight courses across it; find the chance that at least two of them will meet.

Professor Heaton says: "If the men are considered points the chance is 0." [A possible though difficult problem could be made of this one by using instead of men segments of straight lines moving along random secants of a circle, the velocity of the segments all being different. Editor.]

47. Proposed by HENRY HEATON, M. Sc., Atlantic, Iowa.

What is the average length of the chords that may be drawn from one extremity of the major axis of an ellipse to every point of the curve?

Solution by the PROPOSER.

The length of a single chord is

$$[(a-x)^2+y^2]^{\frac{1}{2}}=(1/a)[a^2(a^2-x^2)+b^2(a^2-x^2)]^{\frac{1}{2}}.$$

Put S=distance around the ellipse. Then the required average is $A=$

$$\frac{2}{aS}\int_0^{\frac{1}{2}S}[a^2(a-x)^2-b^2(a^2-x^2)]^{\frac{1}{2}}\,dS=$$

$$\frac{2}{a^2S}\int_{-a}^{+a}\frac{[a^2(a-x)^2+b^2(a^2-x^2)]^{\frac{1}{2}}\,[a^2(a^2-x^2)+b^2x^2]^{\frac{1}{2}}\,dx}{(a^2-x^2)^{\frac{1}{2}}}=$$

$$\frac{2}{a^2S}\int_{-a}^{+a}\frac{[a(a^2+b^2)-(a^2-b^2)x]^{\frac{1}{2}}\,[a^4-(a^2-b^2)x^2]^{\frac{1}{2}}\,dx}{(a+x)^{\frac{1}{2}}}$$

This is readily reducible to elliptic functions of the first and second order, but the expressions I have been able to obtain are involved radicals.

Also solved by *G. B. M. ZERR* and *J. F. SCHEFFER*.

NOTE ON PROBLEM 391

BY LEWIS NEIKIRK, BOULDER, COLORADO.

The man starts at O moving in a *perfectly random* manner. After t seconds suppose him at P and that during the next instant dt he travels through ds to m at an angle θ with the line OP. Let $PM=dr=ds\cos\theta=v\cos\theta dt$, since $ds=vdt$. He will escape from the desert if $\int dr>R$ (the radius) the limits of inte-